Matrix form of linear spectral parameters of the highest splitting method

Pavlov O.

This paper considers the speech signal spectral envelope shape coding theory using a method of linear spectral parameters of the highest splitting (LSP-HS). Advantages of LSP-HS parameter application comparing to classic LSP parameters are specified. It is demonstrated that the direct and inverse transforms of LSP-HS method can be regarded as a certain matrices transform from the LPC coefficients. A formula of synthesized speech signal spectral envelope estimations in space of LSP-HS is derived.

• Encoding shape of the spectral envelope • linear spectral parameters (LSP) • linear spectral pairs (projection LSPr) • linear spectral frequencies (LSF) • linear prediction coefficients (LPC).

В статье рассматривается теория кодирования спектральной огибающей речевого сигнала с использованием метода линейных спектральных параметров наивысшего расщепления (ЛСП-НР). Показаны преимущества ЛСП-НР по сравнению с классическими параметрами метода кодирования с линейным предсказанием. Показано, что прямое и обратное преобразование методом ЛСП-НР можно рассматривать как некоторое матричное преобразование с матрицей, составленной из коэффициентов линейного предсказания. Приведена формула оценки спектральной огибающей синтезированного речевого сигнала непосредственно в пространстве ЛСП-НР.

• кодирование формы спектральной огибающей • линейные спектральные параметры (ЛСП) • линейные спектральные пары (проекции ЛСПр) • линейные спектральные частоты (ЛСЧ) • коэффициенты линейного предсказания (КЛП)

INTRODUCTION

A new original method of speech signal spectral envelope encoding (named as LSP-HS: Linear Spectral Parameter of the Highest Splitting) has been described in previous works [1 – 7]. This method is recommended to use in speech transformation devices of the receiving and transmitting equipments, which are based on the linear prediction (LP) algorithms. The main idea of the method is in that that a characteristic polynomial $A(z)$ of prediction filter of M-order, which represented as one stable polynomial of M-order,

$$A(z)=1-\sum_{i=1}^{M}a_i'z^{-i}=1+\sum_{i=1}^{M}a_iz^{-i}, \qquad (1)$$

is proposed to be represented as M elementary stable normalized polynomials of 1th order

$$A^{vvvv}(z)=1+a_1^{vvvv}z^{-1}, \qquad (2)$$

which are the results of step-by-step splitting of the original polynomial $A(z)$, fig. 1.

The roots of elementary polynomials (2), $A^{vvvv}(z)=1+a_1^{vvvv}z^{-1}$, are the linear spectral *projections* of the highest splitting (LSP-HS).

Arccosine from roots of elementary polynomials (2), $A^{vvvv}(z)=1+a_1^{vvvv}z^{-1}$, are linear spectral *frequencies* of the highest splitting (LSF-HS).

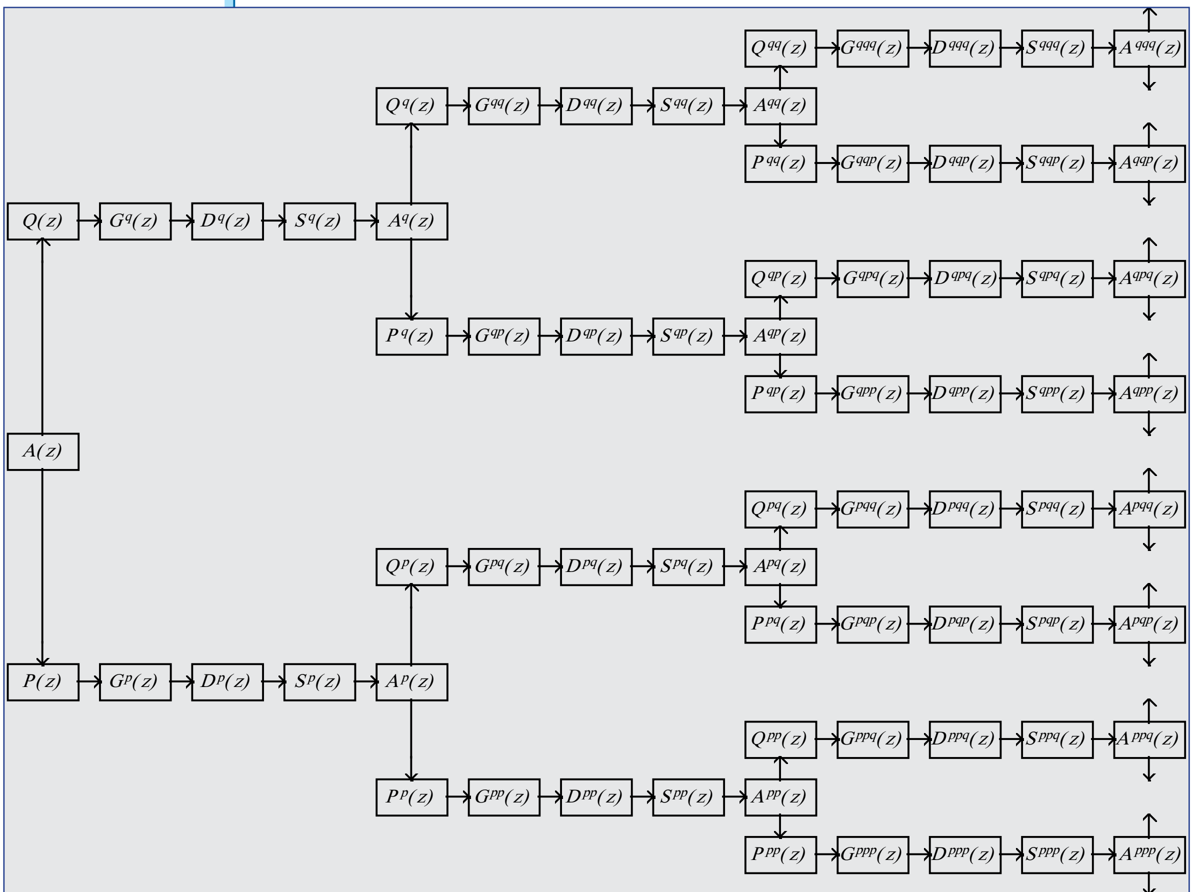

Figure 1 – Structure of LSP-HS inverse transform for stages 1 to 3

1. GOALS OF LSPR-HS AND LSF-HS COMPARED WITH CLASSIC LSP

It was shown that the classical method of linear spectral parameters (frequencies and pairs – LSF, LSPr) are to be only the first stage of regression of the method of linear spectral parameters (projection and frequencies) of the highest splitting – LSF-HS, LSPr-HS.

Transition from classical (first stage of splitting) LSP to the LSP-HS allows for retaining the advantage of the classical method and simultaneously obtaining a number of goals:

1. The process of representing of prediction filter (1), $A(z)=1-\sum_{i=1}^{M}a_i'z^{-i}=1+\sum_{i=1}^{M}a_iz^{-i}$, by LSP-HS is simplified and acquires strict and logically finished form. Roots of elementary polynomials (2), $A^{vvvv}(z)=1+a_1^{vvvv}z^{-1}$, are calculated trivially without using the iterative estimation method since they are equal to the coefficients of a_i^{vvvv} with respect to the sign. Elementary polynomials, in case they are obtained at early stages of splitting, remain invariant with respect to the further stages of split-

ting and do not depend from the value of *M* in (1), $A(z) = 1 - \sum_{i=1}^{M} a_i' z^{-i} = 1 + \sum_{i=1}^{M} a_i z^{-i}$, [1, 2, 7].

2. Elimination of methodological estimation error of the linear spectral parameters, which is proper to the classical method as a result of iteration search of the real interleaving roots of polynomials pair $D^p(x)$ and $D^q(x)$ that, in case of the 10th order linear prediction, have the form of $D^v(x) = x^5 + d_1^v x^4 + d_2^v x^3 + d_3^v x^2 + d_4^v x + d_5^v$ (here the v means any p or q symbol) [1, 2].
3. The algorithm of linear prediction coefficients (LPC) representing in terms of LSP is accelerated [3, 4].
4. The required computational power is distributed between the analyzer of the transmitting side and synthesizer on the receiver side of the speech transformation device more uniformly [5, 6].
5. There exists a simple encoding rule for the chain of upper symbol indexes of coefficients s_i^{vvvv} (where $s_i^{vvvv} \equiv a_i^{vvvv}$), that reflexes a history of coefficient forming in the process stage-wise splitting from (1), $A(z) = 1 - \sum_{i=1}^{M} a_i' z^{-i} = 1 + \sum_{i=1}^{M} a_i z^{-i}$, to (2), $A^{vvvv}(z) = 1 + a_1^{vvvv} z^{-1}$, which allows for making transition to the numerical indexes of coefficients $s_1, \ldots, s_M$, and back to the chain of the upper symbol indexes [7]. Numerical indexes allow for plotting a graph of coefficients of LSP-HS, fig. 2, and determining the appearance of elementary invariant normalized stable first-degree polynomials (2), $A^{vvvv}(z) = 1 + a_1^{vvvv} z^{-1}$, at early stages of splitting for an

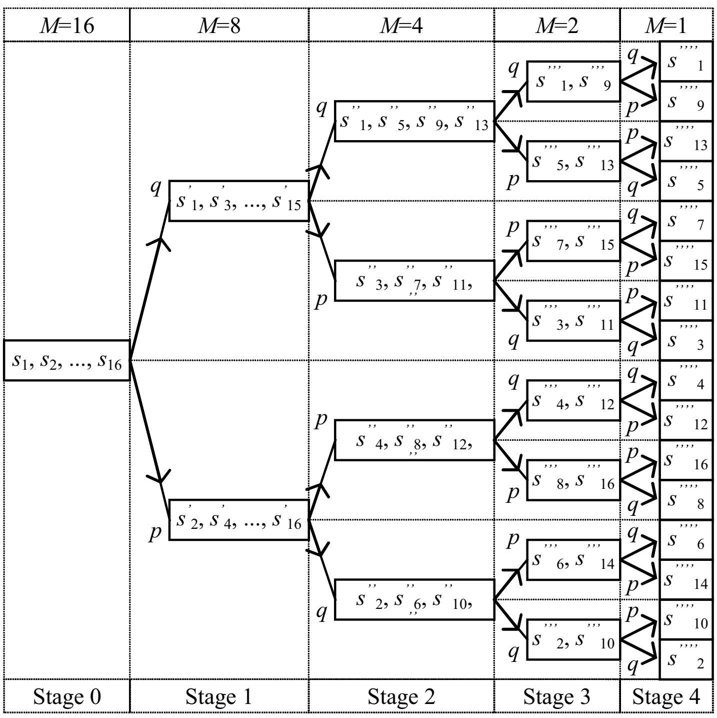

Figure 2 – Graph of formation of LSP-HS coefficients for M = 16
(it is shown how the degree M value is being changed for each stages)

arbitrary value of *M* in (1), $A(z) = 1 - \sum_{i=1}^{M} a_i' z^{-i} = 1 + \sum_{i=1}^{M} a_i z^{-i}$, [7], additionally reducing a number of performed operations.

6. There is a simple stability criterion for the synthesized filter in terms of LSP-HS with numerical indexes in coefficients $s_1,\dots,s_M$, which is invariant for any *M*, [7]:

$$-1 < s_1 < s_2 < s_3 < \dots < s_M < 1.$$

7. LSP-HS provides a smaller error when executing inter-frame interpolation compared to other equivalent parameters, including classical (first stage splitting) LSP, [8].
8. LSP-HS provides a smaller vector quantization error compared to other equivalent parameters, including classical (first stage splitting) LSP, [9].
9. LSP-HS provides lower prediction error of the spectral envelope shape for the speech signal on the basis of known values at the previous frame, [10].

2. LSP-HS INVERSE TRANSFORM

All operations in the inverse transformation of LSP-HS method are linear and LP coefficients, which are recovered from LSP-HS coefficients as a result of the inverse transformation, can be determined by using a generalized linear function of 10 variables: $a_i = f_i(s_1, s_2, \dots, s_{10}) = f_{i,0} + f_{i,1}s_1 + f_{i,2}s_2 + \dots + f_{i,10}s_{10}$, where $s_1 = s_1^{qqqq}$, $s_2 = s_1^{pqqq}$, $s_3 = s_1^{qpqq}$, …, $s_{10} = s_1^{pqqp}$ are ordered LSP-HS coefficients with index numbers corresponding to the enumeration that is started from the one. These coefficients satisfy the rule:

$$-1 < s_1 < s_2 < s_3 < s_4 < s_5 < s_6 < s_7 < s_8 < s_9 < s_{10} < +1. \quad (3)$$

Digital indexation of LSP-HS coefficients are clearly related to their symbolic indexing, which reflects the history of the step-by-step splitting of polynomials in the direct transform of LSP-HS method. This relationship can be defined by the rules:

Each character index q and p is associated with a logical zero and a logical one ($q = 0, p = 1$).

Chain of q and p indices, reflecting the history of the formation of the coefficients resulting from the step-by-step splitting of polynomials in the direct transform of LSP-HS method, is seen as a binary code with the weight of each bit: the weight of the first stage is 2^0, the weight of the second stage is 2^1, and so on.

For convenience, each digital index, which is calculated by the definite binary code, is increased by one that leads to the enumeration of the coefficients that starts from one (as opposed to indexing that starts with zero).

An example of the specified value for the first 16 numbers of coefficients is given in Table 1.

The considered function can be rewritten in a more compact form: $a_i = \sum_{j=0}^{M_A} f_{i,j}s_j$ where $s_0 \equiv 1$ is a formal parameter, which is introduced for convenience, $M_A = 10$ is an order of LP and a degree of the polynomial $A(z) = 1 + \sum_{i=1}^{M_A} a_i z^{-i}$.

Table 1

Relationship of character and numeric indices for $M_A = 16$

Chain of character indices	Chain of binary indices with weights 2021222 3	Digital indices, starts from 0	Number, starts from 1	Chain of character indices	Chain of binary indices with weights 2021222 3	Digital indices, starts from 0	Number, starts from 1
qqqq	0000	0	1	pqqq	1000	1	2
qqqp	0001	8	9	pqqp	1001	9	10
qqpq	0010	4	5	pqpq	1010	5	6
qqpp	0011	12	13	pqpp	1011	13	14
qpqq	0100	2	3	ppqq	1100	3	4
qpqp	0101	10	11	ppqp	1101	11	12
qppq	0110	6	7	pppq	1110	7	8
qppp	0111	14	15	pppp	1111	15	16

The coefficients a_i of the polynomial

$$A(z) = 1 + \sum_{i=1}^{M_A} a_i z^{-i} \qquad (4)$$

can be combined into a vector $\mathbf{A} = \begin{bmatrix} 1 & a_1 & a_2 & \dots & a_{10} \end{bmatrix}^{\mathrm{T}}$, and the value of the complex variable z^{-i}, $0 \le i \le 10$, can be combined into a vector $\mathbf{Z} = \begin{bmatrix} 1 & z^{-1} & z^{-2} & \dots & z^{-10} \end{bmatrix}^{\mathrm{T}}$. Then

$$A(z) = \mathbf{A}^{\mathrm{T}}\mathbf{Z} = \mathbf{Z}^{\mathrm{T}}\mathbf{A}\cdot \qquad (5)$$

By analogy, the result of direct transformation of LPC in LSP-HS space can be combined into a LSP-HS vector, $\mathbf{S} = \begin{bmatrix} 1 & s_1 & s_2 & \dots & s_{10} \end{bmatrix}^{\mathrm{T}}$.

By the linear relationshi $a_i = f_i(s_1, s_2, \dots, s_{10}) = f_{i,0} + f_{i,1}s_1 + f_{i,2}s_2 + \dots + f_{i,10}s_{10}$, $0 \le i \le 10$, we can write the system of ten equations

$$\begin{cases} a_1 = f_1(s_1, s_2, \dots, s_{10}) \\ a_2 = f_2(s_1, s_2, \dots, s_{10}) \\ \dots \\ a_{10} = f_{10}(s_1, s_2, \dots, s_{10}) \end{cases} \text{ in the form of } \begin{cases} f_{1,0} + f_{1,1}s_1 + f_{1,2}s_2 + \dots + f_{1,10}s_{10} = a_1 \\ f_{2,0} + f_{2,1}s_1 + f_{2,2}s_2 + \dots + f_{2,10}s_{10} = a_2 \\ \dots \\ f_{10,0} + f_{10,1}s_1 + f_{10,2}s_2 + \dots + f_{10,10}s_{10} = a_{10} \end{cases}.$$

The 11th trivial equation can be added to this system to bring the system to a quadratic form:

$$\begin{cases} 1 + 0\cdot s_1 + 0\cdot s_2 + \dots + 0\cdot s_{10} = 1 \\ f_{1,0} + f_{1,1}s_1 + f_{1,2}s_2 + \dots + f_{1,10}s_{10} = a_1 \\ f_{2,0} + f_{2,1}s_1 + f_{2,2}s_2 + \dots + f_{2,10}s_{10} = a_2 \\ \dots \\ f_{10,0} + f_{10,1}s_1 + f_{10,2}s_2 + \dots + f_{10,10}s_{10} = a_{10} \end{cases},$$

$$\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \dots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \dots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \dots & f_{2,10} \\ \dots & \dots & \dots & \dots & \dots \\ f_{10,0} & f_{10,1} & f_{10,2} & \dots & f_{10,10} \end{bmatrix} \begin{bmatrix} 1 \\ s_1 \\ s_2 \\ \dots \\ s_{10} \end{bmatrix} = \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \dots \\ a_{10} \end{bmatrix} \text{ or}$$

$$\mathbf{FS}=\mathbf{A}, \text{где } \mathbf{F}=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}, f_{0,0}=1, f_{0,i}=0, 1\le i\le 10 \cdot f_{0,0}=1. \quad (6)$$

Abstracting from the stability criteria in terms of synthesizer filter LSP-HP and rule (3) and acting more formal we can set 11 test vectors LSP-HS: $\mathbf{S}_0=\begin{bmatrix}1\\0\\0\\..\\0\end{bmatrix}$, $\mathbf{S}_1=\begin{bmatrix}1\\1\\0\\..\\0\end{bmatrix}$, $\mathbf{S}_2=\begin{bmatrix}1\\0\\1\\..\\0\end{bmatrix}$, …, $\mathbf{S}_{10}=\begin{bmatrix}1\\0\\0\\..\\1\end{bmatrix}$, which are the columns of the matrix

$$\widetilde{\mathbf{E}}=\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ .. & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}.$$

By using the algorithm of inverse transform from LSP-HS into LPC, $A = \mathrm{rpt}(S)$, for each testing vectors $S_0, S_1, S_2, \dots, S_{10}$ we can find 11 corresponding result LPC vectors $A_0, A_1, A_2, \dots, A_{10}$, which must satisfy the matrix equation (6), $\mathbf{FS}=\mathbf{A}$. Then we can write:

$$\mathbf{A}_0=\begin{bmatrix}1\\a_{0,1}\\a_{0,2}\\ \dots\\a_{0,10}\end{bmatrix}=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1\\0\\0\\..\\0\end{bmatrix}, A_1=\begin{bmatrix}1\\a_{1,1}\\a_{1,2}\\ \dots\\a_{1,10}\end{bmatrix}=$$

$$=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1\\1\\0\\..\\0\end{bmatrix}, \mathbf{A}_2=\begin{bmatrix}1\\a_{2,1}\\a_{2,2}\\ \dots\\a_{2,10}\end{bmatrix}=$$

$$=\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}\begin{bmatrix}1\\0\\1\\..\\0\end{bmatrix}, \dots, \mathbf{A}_{10}=\begin{bmatrix}1\\a_{10,1}\\a_{10,2}\\ \dots\\a_{10,10}\end{bmatrix}=$$

$$= \begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ .. \\ 1 \end{bmatrix},$$

that can be combined into a single matrix equation

$$\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ .. & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ a_{0,1} & a_{1,1} & a_{2,1} & \cdots & a_{10,1} \\ a_{0,2} & a_{1,2} & a_{2,2} & \cdots & a_{10,2} \\ .. & \cdots & \cdots & \cdots & \cdots \\ a_{0,10} & a_{1,10} & a_{2,10} & \cdots & a_{10,10} \end{bmatrix}. \qquad (7)$$

Matrix $\widetilde{\mathbf{E}} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ .. & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}$ in the resulting matrix equation can be reduced to a single form. To do this, each of the columns, except the first, should be subtracted with the first column. In order the matrix equation be not disrupted, similar operations must be executed with columns of the matrix as well, which is the right side of the matrix equation (7). Then we can get:

$$\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ .. & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ a_{0,1} & a_{1,1}-a_{0,1} & a_{2,1}-a_{0,1} & \cdots & a_{10,1}-a_{0,1} \\ a_{0,2} & a_{1,2}-a_{0,2} & a_{2,2}-a_{0,2} & \cdots & a_{10,2}-a_{0,2} \\ .. & \cdots & \cdots & \cdots & \cdots \\ a_{0,10} & a_{1,10}-a_{0,10} & a_{2,10}-a_{0,10} & \cdots & a_{10,10}-a_{0,10} \end{bmatrix}.$$

Finally, the matrix inverse transform LSP-HS $\mathbf{F}$ is defined through the components of the resulting LPC vector in form:

$$\mathbf{F} = \begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ a_{0,1} & a_{1,1}-a_{0,1} & a_{2,1}-a_{0,1} & \dots & a_{10,1}-a_{0,1} \\ a_{0,2} & a_{1,2}-a_{0,2} & a_{2,2}-a_{0,2} & \dots & a_{10,2}-a_{0,2} \\ .. & \dots & \dots & \dots & \dots \\ a_{0,10} & a_{1,10}-a_{0,10} & a_{2,10}-a_{0,10} & \dots & a_{10,10}-a_{0,10} \end{bmatrix}.$$

Values of the matrix of inverse transform LSP-HS $\mathbf{F}$, which have been obtained from calculations for the above technique for $M_A = 10$, are as follows:

	1	0	0	0	0	0	0	0	0	0	0
	0	1	1	1	1	1	1	1	1	1	1
	45	9	7	5	3	1	-1	-3	-5	-7	-9
	0	36	20	8	0	-4	-4	0	8	20	36
	210	84	28	0	-8	-4	4	8	0	-28	-84
F =	0	126	14	-14	-6	6	6	-6	-14	14	126
	210	126	-14	-14	6	6	-6	-6	14	14	-126
	0	84	-28	0	8	-4	-4	8	0	-28	84
	45	36	-20	8	0	-4	4	0	-8	20	-36
	0	9	-7	5	-3	1	1	-3	5	-7	9
	1	1	-1	1	-1	1	-1	1	-1	1	-1

(8)

3. LSP-HS DIRECT TRANSFORM

Similarly, all operations of the direct transform of LPC into LSP-HS can be analyzed and can be shown that the direct transformation $\mathbf{S} = \mathrm{dpt}(\mathbf{A})$ is also linear. Then, by analogy, the coefficients of LSP-HS can be determined using a generalized linear function of 10 variables, $s_i = \phi_i(a_1, a_2, \dots, a_{10}) = \phi_{i,0} + \phi_{i,1}a_1 + \phi_{i,2}a_2 + \dots + \phi_{i,10}a_{10}$, that produce a matrix equation

$$\begin{bmatrix} \phi_{0,0} & \phi_{0,1} & \phi_{0,2} & \dots & \phi_{0,10} \\ \phi_{1,0} & \phi_{1,1} & \phi_{1,2} & \dots & \phi_{1,10} \\ \phi_{2,0} & \phi_{2,1} & \phi_{2,2} & \dots & \phi_{2,10} \\ \dots & \dots & \dots & \dots & \dots \\ \phi_{10,0} & \phi_{10,1} & \phi_{10,2} & \dots & \phi_{10,10} \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \dots \\ a_{10} \end{bmatrix} = \begin{bmatrix} 1 \\ s_1 \\ s_2 \\ \dots \\ s_{10} \end{bmatrix} \text{ or}$$

$$\mathbf{\Phi A} = \mathbf{S} \text{ where } \mathbf{\Phi} = \begin{bmatrix} \phi_{0,0} & \phi_{0,1} & \phi_{0,2} & \dots & \phi_{0,10} \\ \phi_{1,0} & \phi_{1,1} & \phi_{1,2} & \dots & \phi_{1,10} \\ \phi_{2,0} & \phi_{2,1} & \phi_{2,2} & \dots & \phi_{2,10} \\ \dots & \dots & \dots & \dots & \dots \\ \phi_{10,0} & \phi_{10,1} & \phi_{10,2} & \dots & \phi_{10,10} \end{bmatrix}, \ \phi_{0,0} = 1, \ \phi_{0,i} = 0, \ 1 \le i \le 10. \quad (9)$$

	1	0	0	0	0	0	0	0	0	0	0
	-0.998046875	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125	0.001953125
	-0.978515625	0.017578125	0.013671875	0.009765625	0.005859375	0.001953125	-0.001953125	-0.005859375	-0.009765625	-0.013671875	-0.017578125
	-0.890625000	0.070312500	0.039062500	0.015625000	0.000000000	-0.007812500	-0.007812500	0.000000000	0.015625000	0.039062500	0.070312500
	-0.656250000	0.164062500	0.054687500	0.000000000	-0.015625000	-0.007812500	0.007812500	0.015625000	0.000000000	-0.054687500	-0.164062500
	-0.246093750	0.246093750	0.027343750	-0.027343750	-0.011718750	0.011718750	0.011718750	-0.011718750	-0.027343750	0.027343750	0.246093750
Φ =	0.246093750	0.246093750	-0.027343750	-0.027343750	0.011718750	0.011718750	-0.011718750	-0.011718750	0.027343750	0.027343750	-0.246093750
	0.656250000	0.164062500	-0.054687500	0.000000000	0.015625000	-0.007812500	-0.007812500	0.015625000	0.000000000	-0.054687500	0.164062500
	0.890625000	0.070312500	-0.039062500	0.015625000	0.000000000	-0.007812500	0.007812500	0.000000000	-0.015625000	0.039062500	-0.070312500
	0.978515625	0.017578125	-0.013671875	0.009765625	-0.005859375	0.001953125	0.001953125	-0.005859375	0.009765625	-0.013671875	0.017578125
	0.998046875	0.001953125	-0.001953125	0.001953125	-0.001953125	0.001953125	-0.001953125	0.001953125	-0.001953125	0.001953125	-0.001953125

(10)

Solving the equation in a manner similar to the way described above, we can get the matrix of LSP-HS direct conversion, $\mathbf{\Phi}$, defined through the components of the resulting LSP-HS vector, which is:

$$\mathbf{\Phi} = \begin{bmatrix} \phi_{0,0} & \phi_{0,1} & \phi_{0,2} & \cdots & \phi_{0,10} \\ \phi_{1,0} & \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,10} \\ \phi_{2,0} & \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \phi_{10,0} & \phi_{10,1} & \phi_{10,2} & \cdots & \phi_{10,10} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ s_{0,1} & s_{1,1}-s_{0,1} & s_{2,1}-s_{0,1} & \ldots & s_{10,1}-s_{0,1} \\ s_{0,2} & s_{1,2}-s_{0,2} & s_{2,2}-s_{0,2} & \ldots & s_{10,2}-s_{0,2} \\ .. & \ldots & \ldots & \ldots & \ldots \\ s_{0,10} & s_{1,10}-s_{0,10} & s_{2,10}-s_{0,10} & \ldots & s_{10,10}-s_{0,10} \end{bmatrix}.$$

The values of the matrix elements of direct transform of LSP-HS $\mathbf{\Phi}$, which were obtained by computing the above calculations for $M_A = 10$, are as following:

4. BASIS VECTORS OF THE LSP-HS METHOD MATRIX

Matrix of direct (10) and inverse (8) transform of LSP-HS method can be evaluated for any order of M_A. Direct and inverse matrix LSP-HS transformation can be considered as an expansion in the corresponding basis vectors.

The appearance of basic functions of direct and inverse matrix LSP-HS transformation for $M_A = 10$ is shown in Figure 3 and Figure 4.

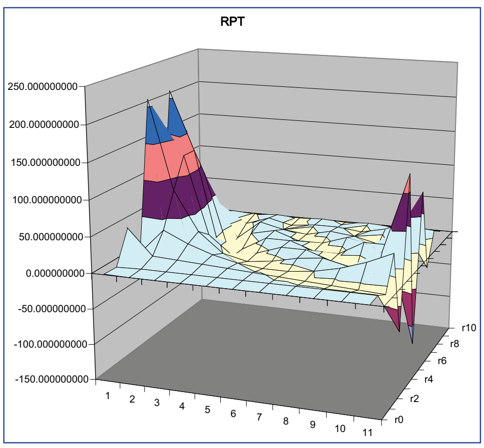

Figure 3 – Basic vectors of the inverse matrix LSP-HS transformation for $M_A = 10$

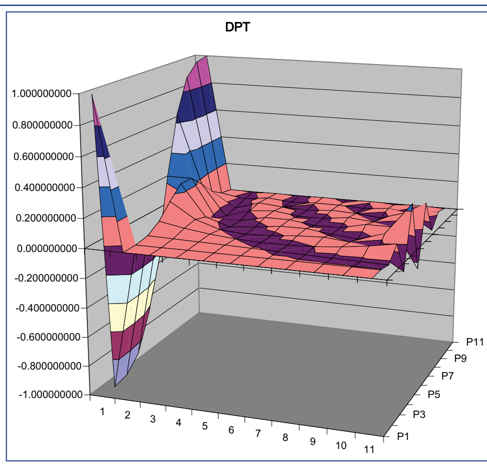

Figure 4 – Basic vectors of the direct matrix LSP-HS transformation for $M_A = 10$

5. ESTIMATION OF THE SPECTRAL ENVELOPE IN THE LSP-HS SPACE

Substituting (6) into (5) we can get an expression for computing the polynomial (4) through the coefficients of direct LSP-HS transformation:

$$A(z) = \mathbf{A}^{\mathrm{T}}\mathbf{Z} = \mathbf{Z}^{\mathrm{T}}\mathbf{A} = (\mathbf{FS})^{\mathrm{T}}\mathbf{Z} = \mathbf{S}^{\mathrm{T}}(\mathbf{F}^{\mathrm{T}}\mathbf{Z}) = (\mathbf{Z}^{\mathrm{T}}\mathbf{F})\mathbf{S}, \qquad (11)$$

which can be rewritten as

$$A(z) = \mathbf{S}^{\mathrm{T}}\widetilde{\mathbf{Z}} = \widetilde{\mathbf{Z}}^{\mathrm{T}}\mathbf{S} \qquad (12)$$

where

$$\widetilde{\mathbf{Z}} = (\mathbf{F}^{\mathrm{T}}\mathbf{Z}) = \begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \\ \cdots \\ z^{-10} \end{bmatrix}, \qquad (13)$$

is a vector rotated by directions of basis vectors of the matrix LSP-HS inverse transform $\mathbf{F}$. Using the substitution, $z = \mathrm{e}^{\mathrm{j}\varphi} = \mathrm{e}^{\mathrm{j}2\pi\omega/\omega_s}$, where ω_S is a circular sampling rate, we can obtain a spectral envelope of the synthesized speech signal for a given frequency ω:

$$K(\mathrm{j}\omega) = \left.\frac{1}{A(z)}\right|_{z=\mathrm{e}^{\mathrm{j}2\pi\omega/\omega_s}} = \left(\begin{bmatrix} 1 \\ s_1 \\ s_2 \\ \cdots \\ s_{10} \end{bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} f_{0,0} & f_{0,1} & f_{0,2} & \cdots & f_{0,10} \\ f_{1,0} & f_{1,1} & f_{1,2} & \cdots & f_{1,10} \\ f_{2,0} & f_{2,1} & f_{2,2} & \cdots & f_{2,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{10,0} & f_{10,1} & f_{10,2} & \cdots & f_{10,10} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1 \\ \mathrm{e}^{-\mathrm{j}2\pi\omega/\omega_s} \\ \mathrm{e}^{-\mathrm{j}4\pi\omega/\omega_s} \\ \cdots \\ \mathrm{e}^{-\mathrm{j}20\pi\omega/\omega_s} \end{bmatrix}\right)\right)^{-1}. \qquad (14)$$

6. RESULTS

It is shown that direct and inverse transforms of LSP-HS method can be regarded as a certain matrix transformation of coefficients of the polynomial (4).

The method for determining the matrix of direct and inverse LSP-HS matrix transformations is described.

Matrix of direct and inverse LSP-HS transforms can be calculated for any degree M_A of the polynomial (4).

Matrixes for direct (10) and inverse (8) LSP-HS transformations for $M_A = 10$ are defined.

The shape of the basis vectors of direct and inverse LSP-HS matrix transformations is shown on fig. 3 and fig. 4.

Formula (14) to estimate the spectral envelope of the synthesized speech signal in LSP-HS space is obtained.

CONCLUSIONS

The method of LSP-HS can be used in matrix form. The matrix form of LSP-HS method makes it possible to employ standard mathematical tools to describe the shape of the spectral envelope of speech signal not only in LPC space, but in the LSP-HS space, which can not be done in the space of classical (first stage splitting) LSP. The matrix form of LSP-HS method allows drawing an alternative structure of the speech synthesis analysis filter, parameters of which are not classic LPC, but the coefficients of LSP-HS space, which reveals the physical meaning of the transition from LPC to LSP in speech coding algorithms.

REFERENCES

1. *O. Pavlov.* Forward P-transformation in Linear Speech Prediction // Radioelectronics and Communications Systems. – **43**(12), 53 (2000).
2. *O. Pavlov.* Simplification of Realization of the Linear Spectral Pairs (Frequencies) Method in the Linear Speech Prediction // in *Proceedings of 3rd International Conference "Digital Signal Processing and Its Application," Moscow, Russia, 2000* (Moscow, 2000), Vol. 3, pp. 128–132.
3. *O. Pavlov.* Fast Algorithm and Graphical Representation of Forward Transformation in the Method of Linear Spectral Frequencies of the Higher Order // in *Proceedings of 3rd International Conference "Digital Signal Processing and Its Application," Moscow, 2000* (Moscow, 2000), Vol. 3, pp. 132–136.
4. *O. Pavlov.* Algorithm of Fast Forward P-Transformation and Peculiarities of Its Mathematics // Radioelectronics and Communications Systems. – **44**(2), 61 (2001).
5. *O. Pavlov.* Reverse P-Transformation in Linear Speech Prediction // Radioelectronics and Communications Systems. – **44**(1), 61 (2001).
6. *O. Pavlov.* Algorithm of Fast Reverse P-Transformation // Radioelectronics and Communications Systems. – **44**(8), 67 (2001).
7. *O. Pavlov,* "Properties of Linear Spectral Frequencies of Higher Orders," in *Radiotekhnika, Vseurk. mezhved. nauch.-teh. sb.* (2001), Vol. 117, pp. 62–64.
8. *O. Pavlov.* Inter-frame interpolation of the spectral envelope of the speech signal in the space of linear spectral frequencies of the highest regression // Radioelectronics and Communications Systems. – April 2008. – Volume 51. – pp. 215 – 223.
9. *O. Pavlov, P. Stasevich, G. Tertichnyi.* Evaluating the effectiveness of the coding of the spectral envelope of speech signals in spaces of linear regression highest spectral parameters using cluster analysis // *in Proceedings of IX All-Ukrainian International Conference* "Signal and Image Processing and Pattern Recognition", UkrObraz'2008, 3—7 November 2008. – Kyiv: IRTC for IT&S, UAsIPPR, 2008. – pp. 189 – 192.
10. *O. Pavlov, K. Gerasymenko, E. Apolonov.* Prediction of the spectral envelope of speech signals in the space of linear spectral parameters of the highest splitting // *in Proceedings of XI All-Ukrainian International Conference* "Signal and Image Processing and Pattern Recognition", UkrObraz'2012, 15—19 Octovber 2012. – Kyiv: IRTC for IT&S and UAsIPPR, 2012. – pp. 137—144.

Сведения об авторе

Павлов Олег Игоревич,
старший преподаватель кафедры теоретических основ радиотехники радиотехнического факультета Национального технического университета Украины «Киевский политехнический институт» (НТУУ «КПИ»).
Область научных интересов: адаптивная обработка сигналов и подавление шума, кодирование источника и обработка речевых сигналов, выбор признаков и распознавание образов, кластерный анализ и векторное квантование, цифровая обработка медицинских сигналов и микробиологических изображений, реализация алгоритмов цифровой обработки сигналов на сигнальных процессорах в реальном времени, быстрые криптографические алгоритмы, специализированные операционные системы реального времени, портативные высокотехнологичные интеллектуальные микроэлектронные устройства на базе сигнальных процессоров.